

Leveraging the Theoretical and Mathematical Underpinnings of a Financial Report

By Charles Hoffman, CPA (Charles.Hoffman@me.com)

Last Revised – October 23, 2018, 2018 (DRAFT)

ABSTRACT: Financial reports are based on the rules of mathematics and the semantics of double-entry bookkeeping. These rules can be leveraged by software engineers creating software intended for creating financial reports. These tools are also useful in creating high-quality accounting process automation functionality. This document points out how I have been able to leverage the theoretical and mathematical underpinnings of a financial report to detect and leverage patterns that exist in financial reports that might not be apparent to most software engineers. These techniques make it possible to disrupt current financial report creation processes.

Copyright (full and complete release of copyright)

All content of this document is placed in the public domain. I hereby waive all claim of copyright in this work. This work may be used, altered or unaltered, in any manner by anyone without attribution or notice to me. To be clear, I am granting full permission to use any content in this work in any way you like. I fully and completely release all my rights to any copyright on this content. If you feel like distributing a copy of this work, you may do so without attribution or payment of any kind. All that said, attribution is appreciated should one feel so compelled. The copyrights of other works referenced by this document are established by the referenced work.

DRAFT

The first recorded use of double-entry accounting was in 1211 AD by a bank in Florence¹. Between 1299 AD and 1300 AD double-entry accounting came of age. In 1494 AD during the Renaissance, Venetian mathematician and Franciscan friar Luca Pacioli published a book, *Summa de arithmetica, geometria. Proportioni et proportionalita* (translated *Sum of Arithmetic, Geometry, Proportion and Proportionality*) documenting this double-entry approach to accounting. Another book documenting double-entry accounting was *Della mercatura e del mercante perfetto*² (translated *Of commerce and the perfect merchant*) which was written by Benedetto Cotrugli in 1458 but it was not published until 1573.

That means it took about 247 to 283 to work out the details of this new system of accounting and document that system so that others could readily copy the system.

Figuring out how to get accounting, reporting, auditing, and analysis to work better; leveraging the nature of the digital environment that now exists; will take some time. Things will evolve over time.

While there are people that say that old-school financial reporting processes need to be improved³ and plenty of very good academic papers that help explain how accounting, reporting, auditing, and analysis in a digital environment might work including *Imagineering Audit 4.0*⁴, *Toward Blockchain-Based Accounting and Assurance*⁵; there is less information on exactly how to make these ideas work in software.

Things can be explained formally such as in a formal academic paper by trained scholars or specialists with deep expertise. This is not a formal academic paper. Things can also be explained informally, in more practical terms based on experimentation of a practitioner trying to figure something out. That is what I am doing in this paper. My hope is that an academic or scholar who has deep knowledge in accounting, math, and knowledge engineering will see what I am trying to explain here and do a better job of explaining how financial statements work in a manner that software engineers can use to build software. This is my best shot.

¹ Geoffrky Alan Lee, *The Development of Italian Bookkeeping 1211–1300*, Wiley, <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-6281.1973.tb00183.x>

² Wikipedia, *Della mercatura e del mercante perfetto*, https://en.wikipedia.org/wiki/Della_mercatura_e_del_mercante_perfetto

³ *Changing Old School Financial Report Creation Processes*, <http://xbrl.squarespace.com/journal/2017/2/14/changing-old-school-financial-report-creation-processes.html>

⁴ Jun Dai and Miklos Vasarhelyi, Rutgers University, *Imagineering Audit 4.0*, <http://aaajournals.org/doi/abs/10.2308/jeta-10494?code=aaan-site>

⁵ Jun Dai and Miklos Vasarhelyi, Rutgers University, *Toward Blockchain-Based Accounting and Assurance*, <http://aaajournals.org/doi/10.2308/isys-51804>

Foundational Mathematical Equation for Double-Entry Accounting

The foundational basis of double-entry accounting is straightforward. Quoting David Ellerman from his paper *The Math of Double-Entry Bookkeeping: Part I (scalars)*⁶:

“Given an equation $w + \dots + x = y + \dots + z$, it is not possible to change just one term in the equation and have it still hold. Two or more terms must be changed.”

And so, the left hand side of the equation “ $w + \dots + x$ ” (the DEBIT side) must always equal the right hand side of the equation “ $y + \dots + z$ ” (the CREDIT side) in double-entry accounting. The reason that double-entry accounting is used, as contrast to single-entry accounting, is double-entry accounting’s capability to detect errors and to distinguish an error from fraud.

Of course, there are a lot of details associated with setting up and operating an accounting system appropriately, but the fundamental feature is that DEBITS must equal CREDITS and if they don’t, then something is up which needs to be investigated and corrected.

Foundational Semantics for Double-Entry Accounting

The foundational semantics of double-entry accounting is the accounting equation⁷:

“Assets = Liabilities and Equity”

The terms “assets”, “liabilities”, and “equity” have very specific and generally well understood meanings in accounting and business. Accounting is referred to as “the language of business”. This high-level accounting equation is broken down into two trees of information. Those two trees are the detailed items which make up a financial report, “Assets” and “Liabilities and Equity”. The semantics reiterates what that foundational equation states and assigns meaning to the numbers that make up those DEBITS and CREDITS.

Empirical evidence gathered shows that there are a finite number of ways used to organize the high-level concepts that make up the three primary financial statements: balance sheet, income statement, cash flow statement. I call these different organizations reporting styles⁸.

⁶ David Ellerman, *The Math of Double-Entry Bookkeeping: Part I (scalars)*, <http://www.ellerman.org/the-math-of-double-entry-bookkeeping-part-i-scalars/>

⁷ Wikipedia, *Accounting Equation*, https://en.wikipedia.org/wiki/Accounting_equation

⁸ Charles Hoffman, *Making the Case for Reporting Styles*, <http://xbrlsite.azurewebsites.net/2017/library/MakingTheCaseForReportingStyles.pdf>

Multi-dimensional double-entry and Triple Entry Accounting

While David Ellerman's articles mentions multi-dimensional double-entry accounting and work in that area by Yuji Ijiri; I am not able to determine if multi-dimensional accounting is the same as triple-entry accounting as used by those creating digital distributed ledgers. I have summarized my thoughts related to distributed ledgers, smart contracts, and XBRL in the document *Distributed Ledgers+Smart Contracts+ XBRL*⁹.

I can show that the account number which tend to be used to represent the chart of accounts of a general ledger are similar to dimensions use to characterize information.

Ledgers and Journals, Stocks and Flows

Another important piece of double-entry accounting is explained well in David Ellerman's article, *The Math of Double-Entry Bookkeeping: Part II (vectors)*, is ledgers and journals¹⁰. Many accountants use the terms "ledger" and "journal" incorrectly. This works the same for general and special ledgers and journals. This is the relationship between a ledger and a journal:

$$\text{Beginning Ledger} + \text{Journal} = \text{Ending Ledger.}$$

Ledgers summarized balances. For example, the general ledger summarizes account balances. Journals record the transactions which make up the changes between ledger balances. Other terms used for the relationship shown above are "roll forward" or "movements" or "stocks and flows". All three of these terms basically explain the following equation:

$$\text{"Beginning balance + Additions - Subtractions = Ending balance"}$$

⁹ Charles Hoffman, *Distributed Ledgers + Smart Contracts + XBRL*, <http://xbrlsite.azurewebsites.net/2018/Library/DistributedLedgersSmartContractsXBRL.pdf>

¹⁰ David Ellerman, *The Math of Double-Entry Bookkeeping: Part II (vectors)*, <http://www.ellerman.org/the-math-of-double-entry-bookkeeping-part-ii-vectors/>

Balance sheet accounts are stocks. Roll forwards of the beginning and ending balances of balance sheet accounts are flows. The income statement is a flow of net income (loss). The cash flow statement is a roll forward of the net change in cash and cash equivalents. The statement of changes in equity is a roll forward of equity accounts.

Many transactions, events, circumstances, and other phenomenon are recorded as transactions in a journal, make their way to a ledger, and then end up in the primary financial statements or within disclosures which detail the line items of the primary financial statements. Much of this information is part of the two trees which make up the roll ups of “Assets” and “Liabilities and Equity”. However, other there are other trees that can make up the complete “forest” of a financial report.

Forests, Trees, Branches, and Leaves

I explained graphs, forests, trees, branches, and leaves in my document *Accounting Process Automation using XBRL*¹¹. Most accountants mistakenly see a chart of accounts as a list. Actually, a chart of accounts is the leaves of a graph or “tree”.

A **tree**¹² is a special type of graph. A tree is what is called an undirected graph because the items in a tree are connected by exactly one path. This is important to understand because it means that trees are safer than other types of graphs which can contain cycles which generally need to be avoided because they can cause problems. For example, one type of cycle is an infinite loop.

A **forest** is a disjointed union of trees. Trees have **branches**. Trees have **leaves**.

A chart of accounts will not have an account or ledger for either “Assets” or “Liabilities and Equity”. “Assets” and “Liabilities and Equity” are branches. For example:

- **Assets**
 - **Current assets**
 - Cash and cash equivalents
 - Cash in bank
 - Petty cash
 - Other cash and cash equivalents
 - Trade accounts receivable
 - Inventories

¹¹ Charles Hoffman, *Accounting Process Automation Using XBRL*, page 5, <http://xbrlsite.azurewebsites.net/2018/Library/AccountingProcessAutomationUsingXBRL.pdf>

¹² Wikipedia, *Tree (Graph Theory)*, [https://en.wikipedia.org/wiki/Tree_\(graph_theory\)](https://en.wikipedia.org/wiki/Tree_(graph_theory))

- ***Noncurrent assets***
 - Property, plant and equipment
 - Goodwill
 - Long-term investments

The items above in bold italics are branches in the tree of assets. “Assets” is the root branch. “Current assets” and “Noncurrent assets” are always branches, you never post transactions to those, they are not accounts and they don’t have ledgers. Depending upon how a chart of accounts is set up something could be a branch or it could be a leaf. For example in the example above, “Cash and cash equivalents” is likely a branch with three leaves for the accounts to which transactions are posted.

Roll Ups

Leaves roll up to their branches; sub-branches roll up to their branches; and these roll up relations culminate at the root of the roll up. For example, the following is a basic roll up:

Balance Sheet [Abstract]	Period [Axis]	
	2018-12-31	2017-12-31
Balance Sheet [Abstract]		
Assets [Roll Up]		
Current Assets [Roll Up]		
Cash and Cash Equivalents	4,000	3,000
Accounts Receivable	2,000	1,000
Inventories	1,000	1,000
Current Assets	7,000	5,000
Noncurrent Assets [Roll Up]		
Property, Plant, and Equipment, Net	6,000	1,000
Noncurrent Assets	6,000	1,000
Assets	13,000	6,000

As can be seen, three current assets roll up to the total of “Current Assets”; one noncurrent asset rolls up to the total “Noncurrent Assets”; and current and noncurrent assets rolls up to the total “Assets”. This can be seen better in the graphic below:

**CC0 1.0 Universal (CC0 1.0)
Public Domain Dedication**

CC0 1.0 Universal (CC0 1.0) Public Domain Dedication <https://creativecommons.org/publicdomain/zero/1.0/>

Label	Rendered	Reported	Calculated	Balance	Decimals	Message
Assets [Roll Up]						
Current Assets [Roll Up]						
Cash and Cash Equivalents	4,000 +	4,000	4,000 DR		INF	
Accounts Receivable	2,000 +	2,000	2,000 DR		INF	
Inventories	1,000 +	1,000	1,000 DR		INF	
Current Assets	7,000 +	7,000	7,000 DR		INF	OK
Noncurrent Assets [Roll Up]						
Property, Plant, and Equipment, Net	6,000 +	6,000	6,000 DR		INF	
Noncurrent Assets	6,000 +	6,000	6,000 DR		INF	OK
Assets	13,000	13,000	13,000 DR		INF	OK

The basic equation for a roll up is “A + B + n = Total”, where some value A plus some value B plus n other values = the roll up total.

Roll Forwards

Changes in ledger balances or the “flows” between a stock at two points in time or the equation “Beginning balance + changes = Ending Balance” is represented in the form of what accountants commonly refer to as a “roll forward” or “movements analysis”. For example, the following is a basic roll forward:

Component: (Network and Table)		
Network	KB - Schedule - Property, Plant and Equipment, Net, Roll Forward (Represented as Line Items/	
Table	Property, Plant and Equipment Roll Forward [Table]	
Reporting Entity [Axis]	1234567890 http://regulator.gov/id	
Reporting Scenario [Axis]	Actual [Member]	
Report Date [Axis]	Reported as of March 18, 2011 [Member]	
Legal Entity [Axis]	Consolidated Entity [Member]	
Unit [Axis]	USD	
	Period [Axis] ▼	
Property, Plant and Equipment Roll Forward [Line Items]	2010-01-01/2010-12-31	2009-01-01/2009-12-31
Land [Roll Forward]		
Land, Beginning Balance	1,000	1,000
Land, Additions	1,000	1,000
Land, Disposals	(1,000)	(1,000)
Land, Translation Difference	0	0
Other Increase (Decrease) in Land	0	0
Land, Ending Balance	1,000	1,000

This shows the mathematical relations more clearly:

Label	Rendered Value	Op	Reported Value	Calculated Value	Balance	Result	Name
▼ Property, Plant and Equipment Roll Forward [Line Items]							gaap:PropertyPlantEquipmentRollForwardLineItems
▼ Land [Roll Forward]							gaap:LandRollForward
Land, Beginning Balance	1,000	+	1,000		Debit		gaap:Land
Land, Additions	1,000	+	1,000		Debit		gaap:LandAdditions
Land, Disposals	(1,000)	-	1,000		Credit		gaap:LandDisposals
Land, Translation Difference	0	+	0		Debit		gaap:LandTranslationDifference
Other Increase (Decrease) in Land	0	+	0		Debit		gaap:OtherIncreaseDecreaseInLand
Land, Ending Balance	1,000	+	1,000	1,000	Debit	Verified	gaap:Land

Adjustment

Adjustments to a ledger balance to correct an error, the equation “Originally stated balance + Adjustments = Restated Balance”. For example, the following is a basic adjustment:

Component: (Network and Table)	
Network	FA - Statement - Changes in Equity, Prior Period Adjustments
Table	Statement of Changes in Equity, Prior Period Adjustments [Table]
Reporting Entity [Axis]	1234567890 http://regulator.gov/id
Legal Entity [Axis]	Consolidated Entity [Member]
Reporting Scenario [Axis]	Actual [Member]
Unit [Axis]	USD
Period [Axis] ▼	
Statement of Changes in Equity, Prior Priod Adjustments [Li...	Report Date [Axis] ▲
	2010-01-01/2010-12-31
Retained Earnings (Accumulated Losses), Prior Period Adjustments [Adjustment]	
Retained Earnings (Accumulated Losses), Originally Stated	Reported as of February 22, 2010 [Member] 2,000
Retained Earnings (Accumulated Losses), Prior Period Errors	Reported as of March 18, 2011 [Member] 500 ¹
Retained Earnings (Accumulated Losses), Changes in Accounting Policies	Reported as of March 18, 2011 [Member] (1,500) ¹
Retained Earnings (Accumulated Losses), Restated	Reported as of March 18, 2011 [Member] 1,000

Again, this shows the mathematical relations more clearly:

Label	Rendered Value	Op	Reported Value	Calculated Value	Balance	Result	Name
Statement of Changes in Equity, Prior Priod Adjustments [Line Items]							gaap:StatementChangesInEquityPriorPriodAdjustmentsLineItems
Retained Earnings (Accumulated Losses), Prior Period Adjustments [Adjustment]							gaap:RetainedEarningsAccumulatedLossesPriorPeriodAdjustmentsAdjustment
Retained Earnings (Accumulated Losses), Originally Stated	2,000	+	2,000		Credit		gaap:RetainedEarningsAccumulatedLosses
Retained Earnings (Accumulated Losses), Prior Period Errors	500	+	500		Credit		gaap:RetainedEarningsAccumulatedLossesPriorPeriodErrors
Retained Earnings (Accumulated Losses), Changes in Accounting	(1,500)	+	-1,500		Credit		gaap:RetainedEarningsAccumulatedLossesChangesInAccountingPolicies
Retained Earnings (Accumulated Losses), Restated	1,000	+	1,000	1,000	Credit	Verified	gaap:RetainedEarningsAccumulatedLosses

Financial Analysis

One of the primary approaches to analyzing an economic entity is to use a discounted cash flow model (DCF) to determine the intrinsic value of that entity. Comparisons across entities and comparisons across time periods is another approach¹³.

SFAS 8 issued by the FASB, page 19, QC23:

"Comparability is not uniformity. For information to be comparable, like things must look alike and different things must look different. Comparability of financial

¹³ Comparing Reporting Styles, Notes on Comparability, <http://xbrl.squarespace.com/journal/2015/11/11/comparing-reporting-styles-notes-on-comparability.html>

information is not enhanced by making unlike things look alike any more than it is enhanced by making like things look different."

Comparability is important and financial reports are supposed to be created to make comparisons and creating things like DCFM possible, not impossible. While there can be issues when trying to compare financial reports, professional financial analysts are adept in creating these comparisons and performing financial analysis.

Financial Report Semantics and Dynamics

Several years ago Rene van Egmond and I summarized our thoughts about the business logic of a financial report and define a conceptual model of such reports in the document *Financial Report Semantics and Dynamics Theory*¹⁴. Over a period of years since that document I have actually created a conceptual model in machine-readable form¹⁵. The machine-readable syntax is pure XBRL.

I have created a report, taxonomy, and all the business rules necessary to provide robust verification to make sure the report is created correctly¹⁶. That example works as expected and proves these ideas in the Pesseract working proof of concept¹⁷ and in commercial software that is made available in XBRL Cloud's Evidence Package¹⁸ which is part of their Clean Score product.

Both the Pesseract and XBRL Cloud software has contributed to significant improvements in the high-level fundamental accounting concept relations of XBRL-based reports that are submitted to the U.S. SEC¹⁹. To a lesser extent, disclosure mechanics quality has improved and quality related to the automated reporting checklist has shown improvement.

While US GAAP XBRL-based reports of about 6,000 public companies have been the primary focus for about five years²⁰, IFRS reports of about 406 foreign issuers have also been tested²¹.

¹⁴ Charles Hoffman and Rene van Egmond, *Financial Report Semantics and Dynamics Theory*, <http://xbrl.azurewebsites.net/2016/Library/Theory-2017-06-26.pdf>

¹⁵ *Conceptual Model of a Digital Financial Report*, <http://xbrl.squarespace.com/conceptual-model/>

¹⁶ You can download the ZIP file here,

http://xbrl.azurewebsites.net/2018/Pesseract/DynamicRules_AllFilesLocal.zip

¹⁷ Understanding Digital Financial Reporting Using Pesseract,

<http://xbrl.squarespace.com/journal/2018/10/14/understanding-digital-financial-reporting-using-pesseract.html>

¹⁸ XBRL Cloud, Clean Score, <https://www.xbrlcloud.com/cleanscore.html>

¹⁹ Quarterly XBRL-based Public Company Financial Report Quality Measurement (Mar 2018),

<http://xbrl.squarespace.com/journal/2018/4/1/quarterly-xbrl-based-public-company-financial-report-quality.html>

²⁰ US GAAP Test Data - 2017 10-Ks, <http://xbrl.squarespace.com/journal/2018/7/28/us-gaap-test-data-2017-10-ks.html>

²¹ Updated List of IFRS Filings, <http://xbrl.squarespace.com/journal/2018/7/14/updated-list-of-ifrs-filings.html>

Putting the Pieces Together

I have published a number of documents trying to show and explain the process of creating a financial report. This is a summary of those documents and what I was trying to achieve with the document.

What I am trying to show in the document *General Ledger Trial Balance to External Financial Report*²², is a basic example of taking the information from a general journal to a general ledger and then ultimately to a financial report using XBRL. The beginning of the process is the accounting system which summarized the information, the end of the process is a human readable and machine readable financial report.

What I was trying to show in the document *INTELLIGENT DIGITAL FINANCIAL REPORTING – PART 4: EXAMPLES AND SAMPLES – COMPREHENSIVE EXAMPLE*²³, is a much more complex financial report which was created using the same techniques as the basic example. Not that this document has several “forests” or disjointed unions of trees meaning that not all the information in this financial report is run through the general ledger trial balance.

What I am trying to show in the blog post *Representing Unlevered Discounted Cash Flow Model Using XBRL*²⁴, is both how information from a financial report is used by financial analysts to analyze economic entities reporting information and that such models can be created using techniques similar to how the financial reports themselves are created.

What I am trying to show in the document *Introducing the Fact Ledger*²⁵, is that intermediate documents or ledgers can be used to summarize information that is used within this process.

What I am trying to show in the document *Blueprint for Creating Zero-Defect XBRL-based Digital Financial Reports*²⁶; is how business rules can be used to automate many of the tasks related to moving information from one step to another, verifying that information is correct and otherwise of high quality so that these processes can be reliable.

What we are trying to show in the document *Putting the Expertise into an XBRL-based Knowledge Based System for Creating Financial Reports*²⁷, are general ideas related to how

²² Charles Hoffman, *General Ledger Trial Balance to External Financial Report*,
<http://xbrl.azurewebsites.net/2018/RoboticFinance/TrialBalanceToReport.pdf>

²³ Charles Hoffman and René van Egmond, *INTELLIGENT DIGITAL FINANCIAL REPORTING – PART 4: EXAMPLES AND SAMPLES – COMPREHENSIVE EXAMPLE*,
http://xbrl.azurewebsites.net/2017/IntelligentDigitalFinancialReporting/Part04_Chapter07.5_ComprehensiveExample.pdf

²⁴ Charles Hoffman, *Representing Unlevered Discounted Cash Flow Model Using XBRL*,
<http://xbrl.squarespace.com/journal/2018/9/4/representing-unlevered-discounted-cash-flow-model-using-xbrl.html>

²⁵ Charles Hoffman and Andrew Noble, *Introducing the Fact Ledger*,
<http://xbrl.azurewebsites.net/2018/Library/IntroductionToTheFactLedger.pdf>

²⁶ Charles Hoffman, *Blueprint for Creating Zero-Defect XBRL-based Digital Financial Reports*,
<http://xbrl.azurewebsites.net/2017/Library/BlueprintForZeroDefectDigitalFinancialReports.pdf>

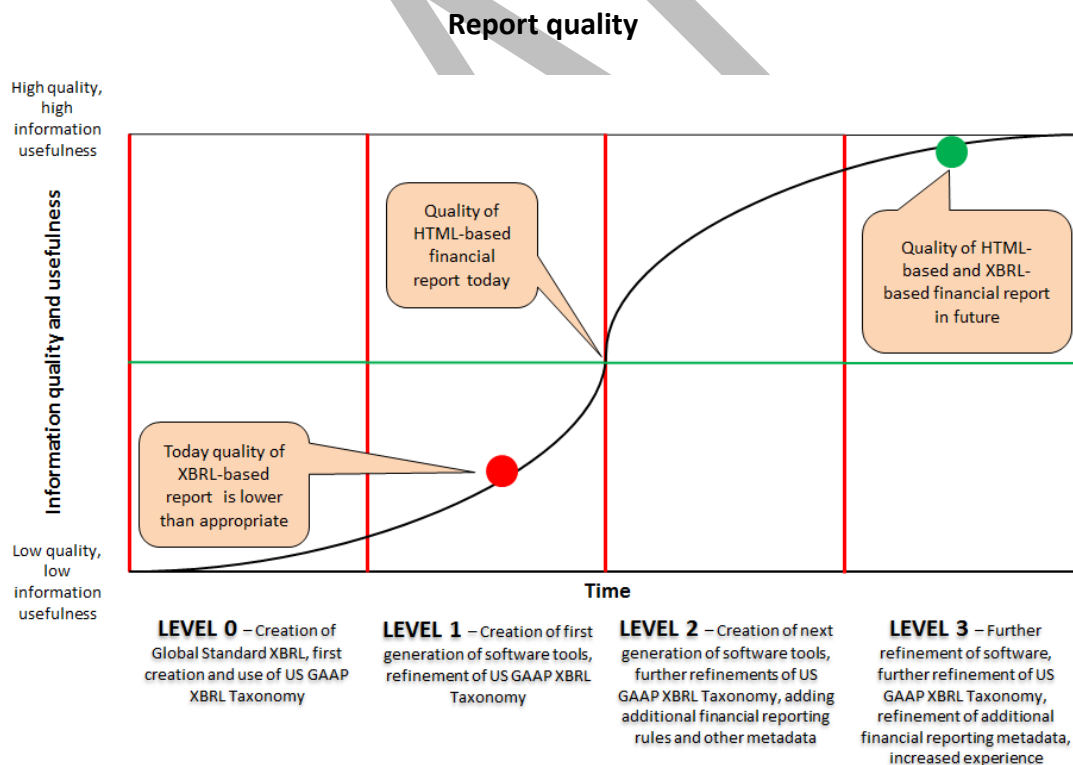
²⁷ Charles Hoffman and Hamed Mousavi, *Putting the Expertise into an XBRL-based Knowledge Based System for Creating Financial Reports*,
<http://pesseraact.azurewebsites.net/PuttingTheExpertiseIntoKnowledgeBasedSystem.pdf>

patterns can be leveraged when creating software that is useful in creating XBRL-based financial reports.

Finally, what I tried to show in the document *Guide to Building an Expert System for Creating Financial Reports*²⁸, are specific techniques we used to create Pesseract²⁹ which is a working proof of concept which we used to prototype and test the ideas in this document. The application can be downloaded and used to better understand these ideas.

Conclusion

One type of practical knowledge is **know-how**; how to accomplish something. This document explains how to accomplish something. The document explains how a working expert system was created which can be useful in the creation of financial reports was created.



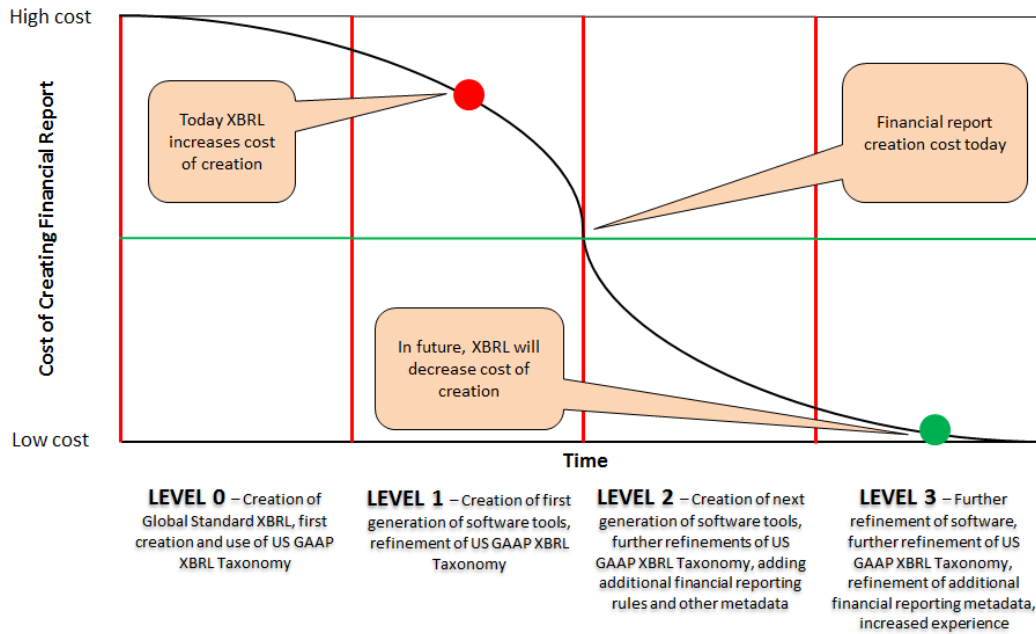
Total cost of creating report

²⁸ Charles Hoffman, *Guide to Building an Expert System for Creating Financial Reports*, <http://xbrl.azurewebsites.net/2018/Library/GuideToBuildingAnExpertSystemForCreatingFinancialReports.pdf>

²⁹ Pesseract, <http://pesseract.azurewebsites.net/>

**CC0 1.0 Universal (CC0 1.0)
Public Domain Dedication**

CC0 1.0 Universal (CC0 1.0) Public Domain Dedication <https://creativecommons.org/publicdomain/zero/1.0/>



Change in the process of constructing financial reports is not going to happen by itself; the changes must be created by accountants and software engineers working together. This could disrupt financial report creation processes as contrast to an incremental innovation.

In order to disrupt³⁰ current approaches to creating financial reports the quality of financial reports has to be as good as the quality of reports today; and the total cost of creating the reports has to be lower than the total cost of creating reports today. Users of the reports may never press the “save as XBRL” button in the application. Disrupting financial reporting has less to do about regulator mandates and more to do with creating a product that is better, faster, and cheaper than today’s financial report creation processes and tools.

Acknowledgements

Most of the ideas in this document come from discussions and feedback that I received over the past 15 or so years from many, many colleagues who are too numerous to list here. That input was critical to shaping the thoughts expressed in this document. Thank you to the entire XBRL community!

³⁰ SEBA Disruption Framework, <http://xbrl.squarespace.com/journal/2018/10/10/seba-technology-disruption-framework.html>