Leveraging the Theoretical and Mathematical Underpinnings of a Financial Report

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ABSTRACT: Financial reports are based on the rules of mathematics and the semantics/logic of double-entry bookkeeping. These rules can be leveraged by software engineers creating software intended for creating financial reports. These tools are also useful in creating high-quality accounting process automation functionality. This document points out how I have been able to leverage the theoretical and mathematical underpinnings of a financial report to detect and leverage patterns that exist in financial reports that might not be apparent to most software engineers. This know-how can be, and eventually will be, documented in the form of a useful methodology. These techniques make it possible to disrupt current financial report creation processes.
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The first recorded use of double-entry accounting was in 1211 AD by a bank in Florence\(^1\). Between 1299 AD and 1300 AD double-entry accounting came of age. In 1494 AD during the Renaissance, Venetian mathematician and Franciscan friar Luca Pacioli published a book, *Summa de arithmetica, geometria. Proportioni et proportionalita* (translated *Sum of Arithmetic, Geometry, Proportion and Proportionality*) documenting this double-entry approach to accounting. Another book documenting double-entry accounting was *Della mercatura e del mercante perfetto*\(^2\) (translated *Of commerce and the perfect merchant*) which was written by Benedetto Cotrugli in 1458 but it was not published until 1573.

That means it took about 247 to 283 years to work out the details of this new system of accounting and document that system so that others could readily copy the system.

Figuring out how to get accounting, reporting, auditing, and analysis to work better; leveraging the nature of the digital environment that now exists; will take some time. Things will evolve over time.

While there are people that say that old-school financial reporting processes need to be improved\(^3\) and plenty of very good academic papers that help explain how accounting, reporting, auditing, and analysis in a digital environment might work including *Imagineering Audit 4.0*\(^4\), *Toward Blockchain-Based Accounting and Assurance*\(^5\); there is less information on exactly how to make these ideas work in software.

Things can be explained formally such as in a formal academic paper by trained scholars or specialists with deep expertise. This is not a formal academic paper. Things can also be explained informally, in more practical terms based on experimentation of a practitioner trying to figure something out. That is what I am doing in this paper. My hope is that an academic or scholar who has deep knowledge in accounting, math, and knowledge engineering will see what I am trying to explain here and do a better job of explaining how financial statements work in a manner that software engineers can use to build software. This is my best shot.

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Foundational Mathematical Equation for Double-Entry Accounting

The foundational basis of double-entry accounting is straightforward. Quoting David Ellerman from his paper *The Math of Double-Entry Bookkeeping: Part I (scalars)*:

“Given an equation \( w + ... + x = y + ... + z \), it is not possible to change just one term in the equation and have it still hold. Two or more terms must be changed.”

And so, the left hand side of the equation “\( w + ... + x \)” (the DEBIT side) must always equal the right hand side of the equation “\( y + ... + z \)” (the CREDIT side) in double-entry accounting. The reason that double-entry accounting is used, as contrast to single-entry accounting, is double-entry accounting’s capability to detect errors and to distinguish an error from fraud.

Of course, there are a lot of details associated with setting up and operating an accounting system appropriately, but the fundamental feature is that DEBITS must equal CREDITS and if they don’t, then something is up which needs to be investigated and corrected.

Foundational Semantics for Double-Entry Accounting

The foundational semantics of double-entry accounting is the accounting equation:

“Assets = Liabilities and Equity”

The terms “assets”, “liabilities”, and “equity” have very specific and generally well understood meanings in accounting and business. Accounting is referred to as “the language of business”. This high-level accounting equation is broken down into two trees of information. Those two trees are the detailed items which make up a financial report, “Assets” and “Liabilities and Equity”. The semantics reiterates what that foundational equation states and assigns meaning to the numbers that make up those DEBITS and CREDITS.

Empirical evidence gathered shows that there are a finite number of ways used to organize the high-level concepts that make up the three primary financial statements: balance sheet, income statement, cash flow statement. I call these different organizations reporting styles.

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Multi-dimensional double-entry and Triple Entry Accounting

While David Ellerman’s articles mention multi-dimensional double-entry accounting and work in that area by Yuji Ijiri; I am not able to determine if multi-dimensional accounting is the same as triple-entry accounting as used by those creating digital distributed ledgers. I have summarized my thoughts related to distributed ledgers, smart contracts, and XBRL in the document Distributed Ledgers+Smart Contracts+ XBRL. I can show that the account number which tend to be used to represent the chart of accounts of a general ledger are similar to dimensions use to characterize information.

Ledgers and Journals, Stocks and Flows

Another important piece of double-entry accounting is explained well in David Ellerman’s article, The Math of Double-Entry Bookkeeping: Part II (vectors), is ledgers and journals. Many accountants use the terms “ledger” and “journal” incorrectly. This works the same for general and special ledgers and journals. This is the relationship between a ledger and a journal:

Ledgers summarized balances. For example, the general ledger summarizes account balances.

Journals record the transactions which make up the changes between ledger balances. Other terms used for the relationship shown above are “roll forward” or “movements” or “stocks and flows”. All three of these terms basically explain the following equation:

“Beginning balance + Additions - Subtractions = Ending balance”

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Balance sheet accounts are stocks. Roll forwards of the beginning and ending balances of balance sheet accounts are flows. The income statement is a flow of net income (loss). The cash flow statement is a roll forward of the net change in cash and cash equivalents. The statement of changes in equity is a roll forward of equity accounts.

Many transactions, events, circumstances, and other phenomenon are recorded as transactions in a journal, make their way to a ledger, and then end up in the primary financial statements or within disclosures which detail the line items of the primary financial statements. Much of this information is part of the two trees which make up the roll ups of “Assets” and “Liabilities and Equity”. However, other there are other trees that can make up the complete “forest” of a financial report.

**Forests, Trees, Branches, and Leaves**

I explained graphs, forests, trees, branches, and leaves in my document *Accounting Process Automation using XBRL*[^11]. Most accountants mistakenly see a chart of accounts as a list. Actually, a chart of accounts is the leaves of a graph or “tree”.

A **tree**[^12] is a special type of graph. A tree is what is called an undirected graph because the items in a tree are connected by exactly one path. This is important to understand because it means that trees are safer than other types of graphs which can contain cycles which generally need to be avoided because they can cause problems. For example, one type of cycle is an infinite loop.

A **forest** is a disjointed union of trees. Trees have **branches**. Trees have **leaves**.

A chart of accounts will not have an account or ledger for either “Assets” or “Liabilities and Equity”. “Assets” and “Liabilities and Equity” are branches. For example:

- **Assets**
  - **Current assets**
    - Cash and cash equivalents
      - Cash in bank
      - Petty cash
      - Other cash and cash equivalents
    - Trade accounts receivable
    - Inventories

Noncurrent assets

- Property, plant and equipment
- Goodwill
- Long-term investments

The items above in bold italics are branches in the tree of assets. “Assets” is the root branch. “Current assets” and “Noncurrent assets” are always branches, you never post transactions to those, they are not accounts and they don’t have ledgers. Depending upon how a chart of accounts is set up something could be a branch or it could be a leaf. For example in the example above, “Cash and cash equivalents” is likely a branch with three leaves for the accounts to which transactions are posted.

Roll Ups

Leaves roll up to their branches; sub-branches roll up to their branches; and these roll up relations culminate at the root of the roll up. For example, the following is a basic roll up:

<table>
<thead>
<tr>
<th>Balance Sheet [Abstract]</th>
<th>Period [Axis]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2018-12-31</td>
</tr>
<tr>
<td><strong>Assets [Roll Up]</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Current Assets [Roll Up]</strong></td>
<td></td>
</tr>
<tr>
<td>Cash and Cash Equivalents</td>
<td>4,000</td>
</tr>
<tr>
<td>Accounts Receivable</td>
<td>2,000</td>
</tr>
<tr>
<td>Inventories</td>
<td>1,000</td>
</tr>
<tr>
<td>Current Assets</td>
<td><strong>7,000</strong></td>
</tr>
<tr>
<td><strong>Noncurrent Assets [Roll Up]</strong></td>
<td></td>
</tr>
<tr>
<td>Property, Plant, and Equipment, Net</td>
<td></td>
</tr>
<tr>
<td>Noncurrent Assets</td>
<td><strong>6,000</strong></td>
</tr>
<tr>
<td><strong>Assets</strong></td>
<td><strong>13,000</strong></td>
</tr>
</tbody>
</table>

As can be seen, three current assets roll up to the total of “Current Assets”; one noncurrent asset rolls up to the total “Noncurrent Assets”; and current and noncurrent assets rolls up to the total “Assets”. This can be seen better in the graphic below:
The basic equation for a roll up is “A + B + n = Total”, where some value A plus some value B plus n other values = the roll up total.

**Roll Forwards**

Changes in ledger balances or the “flows” between a stock at two points in time or the equation “Beginning balance + changes = Ending Balance” is represented in the form of what accountants commonly refer to as a “roll forward” or “movements analysis”. For example, the following is a basic roll forward:

This shows the mathematical relations more clearly:
Adjustment

Adjustments to a ledger balance to correct an error, the equation “Originally stated balance + Adjustments = Restated Balance”. For example, the following is a basic adjustment:

<table>
<thead>
<tr>
<th>Reporting Entity [Axis]</th>
<th>1234567890 <a href="http://regulator.gov/id">http://regulator.gov/id</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>Legal Entity [Axis]</td>
<td>Consolidated Entity [Member]</td>
</tr>
<tr>
<td>Reporting Scenario [Axis]</td>
<td>Actual [Member]</td>
</tr>
<tr>
<td>Unit [Axis]</td>
<td>USD</td>
</tr>
</tbody>
</table>

Statement of Changes in Equity, Prior Period Adjustments [Li.]

<table>
<thead>
<tr>
<th>Retained Earnings (Accumulated Losses), Originally Stated</th>
<th>Reported as of February 22, 2010 [Member]</th>
<th>2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retained Earnings (Accumulated Losses), Prior Period Errors</td>
<td>Reported as of March 18, 2011 [Member]</td>
<td>500</td>
</tr>
<tr>
<td>Retained Earnings (Accumulated Losses), Changes in Accounting Policies</td>
<td>Reported as of March 18, 2011 [Member]</td>
<td>1,500</td>
</tr>
<tr>
<td>Retained Earnings (Accumulated Losses), Restated</td>
<td>Reported as of March 18, 2011 [Member]</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Again, this shows the mathematical relations more clearly:

<table>
<thead>
<tr>
<th>Label</th>
<th>Rendered Value</th>
<th>Reported Value</th>
<th>Calculated Value</th>
<th>Balance</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Financial Analysis

One of the primary approaches to analyzing an economic entity is to use a discounted cash flow model (DCF M) to determine the intrinsic value of that entity. Comparisons across entities and comparisons across time periods is another approach.¹³

SFAS 8 issued by the FASB, page 19, QC23:

"Comparability is not uniformity. For information to be comparable, like things must look alike and different things must look different. Comparability of financial

information is not enhanced by making unlike things look alike any more than it is enhanced by making like things look different."

Comparability is important and financial reports are supposed to be created to make comparisons and creating things like DCFM possible, not impossible. While there can be issues when trying to compare financial reports, professional financial analysts are adept in creating these comparisons and performing financial analysis.

Financial Report Semantics and Dynamics

Several years ago Rene van Egmond and I summarized our thoughts about the business logic of a financial report and define a conceptual model of such reports in the document Financial Report Semantics and Dynamics Theory\textsuperscript{14}. Over a period of years since that document I have actually created a conceptual model in machine-readable form\textsuperscript{15}. The machine-readable syntax is pure XBRL.

I have created a report, taxonomy, and all the business rules necessary to provide robust verification to make sure the report is created correctly\textsuperscript{16}. That example works as expected and proves these ideas in the Pessacter working proof of concept\textsuperscript{17} and in commercial software that is made available in XBRL Cloud’s Evidence Package\textsuperscript{18} which is part of their Clean Score product.

Both the Pessacter and XBRL Cloud software has contributed to significant improvements in the high-level fundamental accounting concept relations of XBRL-based reports that are submitted to the U.S. SEC\textsuperscript{19}. To a lesser extent, disclosure mechanics quality has improved and quality related to the automated reporting checklist has shown improvement.

While US GAAP XBRL-based reports of about 6,000 public companies have been the primary focus for about five years\textsuperscript{20}, IFRS reports of about 406 foreign issuers have also been tested\textsuperscript{21}.

\begin{itemize}
  \item \textsuperscript{15} Conceptual Model of a Digital Financial Report, \url{http://xbrl.squarespace.com/conceptual-model/}
  \item \textsuperscript{16} You can download the ZIP file here, \url{http://xbrlsite.azurewebsites.net/2018/Pesseract/DynamicRules_AllFilesLocal.zip}
  \item \textsuperscript{17} Understanding Digital Financial Reporting Using Pessacter, \url{http://xbrl.squarespace.com/journal/2018/10/14/understanding-digital-financial-reporting-using-pesseract.html}
  \item \textsuperscript{18} XBRL Cloud, Clean Score, \url{https://www.xbrlcloud.com/cleanscore.html}
  \item \textsuperscript{20} US GAAP Test Data - 2017 10-Ks, \url{http://xbrl.squarespace.com/journal/2018/7/28/us-gaap-test-data-2017-10-ks.html}
  \item \textsuperscript{21} Updated List of IFRS Filings, \url{http://xbrl.squarespace.com/journal/2018/7/14/updated-list-of-ifrs-filings.html}
\end{itemize}
Comparison of Reporting Schemes

I put together a comparison of six different financial reporting schemes. The side-by-side comparison allows you to compare and contrast different reporting schemes to see the similarities and differences between the high-level concepts of these reporting schemes. Some of these reporting schemes have been represented using XBRL, others have not.

What might seem striking to non-accountants, maybe even to accountants, is the similarity between the reporting schemes at a high level. Clearly all reporting schemes have the accounting equation at the highest level: Assets = Liabilities and Equity. The high-level concepts provide the breakdowns of Assets, Liabilities, and Equity used by that reporting scheme.

What one recognizes if they understand the leverage that patterns provide and they understand how computers work is the leverage that would be provided by a meta-meta-model of a financial report. Such a meta-model of a financial report enables the efficient creation of software that is approachable and easy for professional accountants to use.

Packaging ideas into a Documented Methodology

Per Wikipedia, a methodology is the systematic, theoretical analysis of the methods applied to a field of study. It comprises the theoretical analysis of the body of methodology and principles associated with a branch of knowledge. Typically, it encompasses concepts such as paradigm, theoretical model, phases and quantitative or qualitative techniques.

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A methodology does not set out to provide a solution. A methodology is, therefore, not the same as a method. Instead, a methodology offers the theoretical underpinnings for understanding which method, or set of methods, or so called "best practices" can be applied to specific case, for example, to calculating a specific result. A best practice is a method or technique that has been generally accepted as superior to any alternatives because it produces results that are superior to those achieved by other means or because it has become a standard way of doing things, e.g., a standard way of complying with legal or ethical requirements.

A meta-meta model and documented methodology will help those attempting to implement XBRL-based reporting to not have to “re-invent the wheel”. This is particularly important if XBRL were to be adopted by individual companies internally. There is no way the typical individual organization would tolerate the complexity which they currently are exposed to when attempting to implement XBRL for financial reporting.

For XBRL to be used for accounting process automation and the transformation of financial reporting processes, a standard meta-meta model and documented methodology are essential.

Putting the Pieces Together

I have published a number of documents trying to show and explain the process of creating a financial report. This is a summary of information from those documents and what I was trying to achieve with the document. Right now I can say that I do have a method for creating high-quality XBRL-based digital financial reports. I cannot say that I have a well-documented methodology.

What I am trying to show in the document General Ledger Trial Balance to External Financial Report, is a basic example of taking the information from a general journal to a general ledger and then ultimately to a financial report using XBRL. The beginning of the process is the accounting system which summarized the information, the end of the process is a human readable and machine readable financial report.

What I was trying to show in the document INTELLIGENT DIGITAL FINANCIAL REPORTING – PART 4: EXAMPLES AND SAMPLES – COMPREHENSIVE EXAMPLE, is a much more complex financial report which was created using the same techniques as the basic example. Not that this document has several “forests” or disjointed unions of trees meaning that not all the information in this financial report is run through the general ledger trial balance.

What I am trying to show in the blog post *Representing Unlevered Discounted Cash Flow Model Using XBRL*\(^\text{28}\), is both how information from a financial report is used by financial analysts to analyze economic entities reporting information and that such models can be created using techniques similar to how the financial reports themselves are created.

What I am trying to show in the document *Introducing the Fact Ledger*\(^\text{29}\), is that intermediate documents or ledgers can be used to summarize information that is used within this process.

What I am trying to show in the document *Blueprint for Creating Zero-Defect XBRL-based Digital Financial Reports*\(^\text{30}\); is how business rules can be used to automate many of the tasks related to moving information from one step to another, verifying that information is correct and otherwise of high quality so that these processes can be reliable.

What we are trying to show in the document *Putting the Expertise into an XBRL-based Knowledge Based System for Creating Financial Reports*\(^\text{31}\), are general ideas related to how patterns can be leveraged when creating software that is useful in creating XBRL-based financial reports.

Finally, what I tried to show in the document *Guide to Building an Expert System for Creating Financial Reports*\(^\text{32}\), are specific techniques we used to create Pesseract\(^\text{33}\) which is a working proof of concept which we used to prototype and test the ideas in this document. The application can be downloaded and used to better understand these ideas.

My next step is to turn all of this information into a documented methodology that is easy to understand.

**Conclusion**

One type of practical knowledge is **know-how**; how to accomplish something. This document explains at a high level how to accomplish something. The document explains how a working expert system was created which can be useful in the creation of financial reports was created. I believe that that current approaches to creating financial reports which tend to be painful,
monotonous, onerous, and even grueling, barbaric, and gruesome. Once professional accountants see new approaches, then the reality of the current approach will become apparent.

I am not talking about an incremental or sustaining improvement to existing approaches to creating financial reports. Rather, I am proposing that what will occur will be a disruptive or transformative change; perhaps even considered a foundational change.

Change in the process of constructing financial reports is not going to happen by itself; the changes must be created by accountants and software engineers working together. This could disrupt financial report creation processes as contrast to an incremental innovation.

The diagrams below show what I call LEVEL 0, LEVEL 1, LEVEL 2, and LEVEL 3 of the evolution of this change over time.

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In order to disrupt\textsuperscript{36} current approaches to creating financial reports the quality of financial reports has to be as good as the quality of reports today; and the total cost of creating the reports has to be lower than the total cost of creating reports today. Users of the reports may never press the “save as XBRL” button in the application. Disrupting financial reporting has less to do about regulator mandates and more to do with creating a product that is better, faster, and cheaper than today’s financial report creation processes and tools.

**Acknowledgements**

Most of the ideas in this document come from discussions and feedback that I received over the past 15 or so years from many, many colleagues who are too numerous to list here. That input was critical to shaping the thoughts expressed in this document. Thank you to the entire XBRL community!