

Accounting Equation

Building on the double-entry accounting model

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“I skate to where the puck is going to be, not where it has been.” *Wayne Gretzky*,
legendary Canadian hockey star

Executive summary:

- The double-entry accounting model and the accounting equation form the core shell for all financial reporting schemes.
- This document enhances the accounting equation in two ways. First, it puts these definitions in machine-readable form¹. Second, it puts the elements in context by explicitly showing the associations between the defined elements.

¹ Human-readable and machine-readable documentation, <http://xbrlsite.azurewebsites.net/2019/Core/core-sfac6/>

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The accounting equation and double-entry accounting are the foundation of financial reporting schemes.

This document enhances the accounting equation in two ways. First, it puts these definitions in machine-readable form. Second, it puts the elements in context by showing the associations between the concepts in machine-readable form.

Double-entry Accounting.

Single-entry accounting is how ‘everyone’ would do accounting. In fact, that is how accounting was done for about 4,000 years before double-entry accounting was invented. Double-entry accounting was the invention of medieval merchants and was first documented by the Italian mathematician and Franciscan Friar Luca Pacioli² in 1494. The section related to double-entry accounting was translated into English in 1914³. Double entry accounting is based on well-known mathematics construction from undergraduate algebra⁴.

Double-entry accounting adds an additional important property to the accounting system, that of a clear strategy to identify errors and to remove the errors from the system. Even better, double-entry accounting has a side effect of clearly firewalling errors as either accident or fraud. This then leads to an audit strategy. Double-entry accounting is how professional accountants do accounting.

Which came first, double-entry accounting or the enterprise? It is hard to overstate the impact of double-entry accounting on the evolution of the complex global enterprise⁵.

Foundational Mathematical Equation for Double-Entry Accounting

The foundational basis of double-entry accounting is straightforward. Quoting David Ellerman from his paper *The Math of Double-Entry Bookkeeping: Part I (scalars)*⁶:

“Given an equation $w + \dots + x = y + \dots + z$, it is not possible to change just one term in the equation and have it still hold. Two or more terms must be changed.”

² Wikipedia, *Luca Pacioli*, https://en.wikipedia.org/wiki/Luca_Pacioli

³ J. B. Geijsbeek, *Ancient Double-Entry Bookkeeping*, <https://archive.org/details/ancientdoubleent00geij/page/n3>

⁴ David Ellerman, *Mathematics Magazine*, *The Math of Double-Entry Bookkeeping*, page 226, http://www.ellerman.org/wp-content/uploads/2012/12/DEB-Math-Mag.CV_.pdf

⁵ Ian Grigg, *Triple Entry Accounting*, https://iang.org/papers/triple_entry.html

⁶ David Ellerman, *The Math of Double-Entry Bookkeeping: Part I (scalars)*, <http://www.ellerman.org/the-math-of-double-entry-bookkeeping-part-i-scalars/>

And so, the left-hand side of the equation “w + ... + x” (the DEBIT side) must always equal the right-hand side of the equation “y + ... + z” (the CREDIT side) in double-entry accounting. The reason that double-entry accounting is used, as contrast to single-entry accounting, is double-entry accounting’s capability to detect errors and to distinguish an error from fraud.

Of course, there are a lot of details associated with setting up and operating an accounting system appropriately, but the fundamental feature is that DEBITS must equal CREDITS and if they don’t, then something is up which needs to be investigated and corrected.

If you desire to learn more about double-entry accounting, see Colin Dodd’s rap song, Debit Credit Theory (Accounting Rap Song)⁷.

The Accounting Equation: Framework for Financial Accounting

While the model “Debits = Credits” or the notion of basically using two single entry ledgers and synchronizing them to detect errors or fraud is useful; additional power is provided to double-entry accounting via the accounting equation⁸ which is:

$$\text{“Assets = Liabilities + Equity”}$$

The accounting equation within the double-entry accounting is the fundamental basis for financial accounting. By definition, every financial reporting scheme⁹ has this high-level model at its core.

Ledgers and Journals, Stocks and Flows

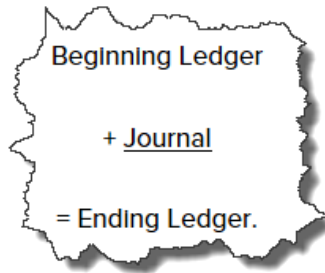
Another important piece of double-entry accounting is explained well in David Ellerman’s article, *The Math of Double-Entry Bookkeeping: Part II (vectors)*, is ledgers and journals¹⁰. Many accountants use the terms “ledger” and “journal” incorrectly. This works the same for general and special ledgers and journals. This is the relationship between a ledger and a journal:

⁷ YouTube, *Colin Dodd’s rap song, Debit Credit Theory (Accounting Rap Song)*, <https://www.youtube.com/watch?v=j71Kmxv7smk>

⁸ Wikipedia, Accounting Equation, https://en.wikipedia.org/wiki/Accounting_equation

⁹ Charles Hoffman, CPA, *Comparison of Financial Reporting Schemes High Level Concepts*, <http://xbrlsite.azurewebsites.net/2018/Library/ReportingSchemes-2018-12-30.pdf>

¹⁰ David Ellerman, *The Math of Double-Entry Bookkeeping: Part II (vectors)*, <http://www.ellerman.org/the-math-of-double-entry-bookkeeping-part-ii-vectors/>



Ledgers summarized balances. For example, the general ledger summarizes account balances.

Journals record the transactions which make up the changes between ledger balances. Other terms used for the relationship shown above are “roll forward” or “movements” or “stocks and flows” or “account analysis”. All three of these terms basically explain the following equation:

$$\text{“Beginning balance + Additions - Subtractions = Ending balance”}$$

Balance sheet accounts are stocks. Roll forwards of the beginning and ending balances of balance sheet accounts are flows. The income statement is a flow of net income (loss). The cash flow statement is a roll forward of the net change in cash and cash equivalents. The statement of changes in equity is a roll forward of equity accounts.

Many transactions, events, circumstances, and other phenomenon are recorded as transactions in a journal, make their way to a ledger, and then end up in the primary financial statements or within disclosures which detail the line items of the primary financial statements. Much of this information is part of the two trees which make up the roll ups of “Assets” and “Liabilities and Equity”. However, other there are other trees that can make up the complete “forest” of a financial report. For more information about the “forest” and the “trees” of a financial report, see the document *Leveraging the Theoretical and Mathematical Underpinnings of a Financial Report*¹¹. That document also has some good information related to triple-entry accounting which I am not going to get into here.

As pointed out in the document *General Ledger Trial Balance to External Financial Report*¹², each balance sheet line item has a roll forward. While perhaps not reported externally, these roll forwards can be quite helpful internally to verify that a financial report has been created correctly.

¹¹ Charles Hoffman, CPA, *Leveraging the Theoretical and Mathematical Underpinnings of a Financial Report*, <http://xbrlsite.azurewebsites.net/2018/Library/TheoreticalAndMathematicalUnderpinningsOfFinancialReport.pdf#page=6>

¹² Charles Hoffman, CPA, *General Ledger Trial Balance to External Financial Report*, <http://xbrlsite.azurewebsites.net/2018/RoboticFinance/TrialBalanceToReport.pdf>

The Accounting Equation

The accounting equation¹³ defines three core elements of a financial report:

- Assets
- Liabilities
- Equity

The accounting equation defines three terms and provides the relations between the three terms:

$$\text{Assets} = \text{Liabilities} + \text{Equity}$$

The following logical system provides the same information in machine-readable and human-readable form.

Describing a Logical System

A **logical system**¹⁴ (logical theory) enables a community of stakeholders trying to achieve a specific goal or objective or a range of goals/objectives to agree on important common models, structures, and statements for capturing meaning or representing a shared understanding of and knowledge in some universe of discourse.

A logical system or logical theory is made up of a set of **models, structures, terms, associations, assertions, and facts**. In very simple terms,

- **Logical theory:** A logical theory is a set of *models* that are consistent with that logical theory.
- **Model:** A model is a set of *structures*. A model is an interpretation of a theory.
- **Structure:** A structure is a set of *statements* which describe the structure.
- **Statement:** A statement is a proposition, claim, assertion, belief, idea, or fact about or related to the universe of discourse. There are four broad categories of statements:
 - **Terms:** Terms are statements that define ideas used by the logical theory such as “assets”, “liabilities”, and “equity”.
 - **Associations:** Associations are statements that describe permissible interrelationships between the terms such as “assets is part-of the balance sheet” or “operating expenses is a type-of expense” or “assets = liabilities +

¹³ Wikipedia, *Accounting Equation*, https://en.wikipedia.org/wiki/Accounting_equation

¹⁴ Charles Hoffman, CPA, *Understanding and Expressing Logical Systems*, <http://xbrl.squarespace.com/journal/2019/9/25/understanding-and-expressing-logical-systems.html>

equity” or “an asset is a ‘debit’ and is ‘as of’ a specific point in time and is always a monetary numeric value”.

- **Assertions:** Assertions are statements that describe what tend to be IF...THEN...ELSE types of relationships such as “IF the economic entity is a not-for-profit THEN net assets = assets - liabilities; ELSE assets = liabilities + equity”
- **Facts:** Facts are statements about the numbers and words that are provided by an economic entity within their financial report. For example, “assets for the consolidated legal entity Microsoft as of June 20, 2017 was \$241,086,000,000 expressed in US dollars and rounded to the nearest millions of dollars.

A logical system can have high to low **precision** and high to low **coverage**. *Precision* is a measure of how precisely the information within a logical system has been represented as contrast to reality for the universe of discourse. *Coverage* is a measure of how completely information in a logical system has been represented relative to the reality for a universe of discourse.

Logical System Represented Using XBRL

The following is a summary of the logical model of the accounting equation in both machine-readable and human-readable terms.

TERMS^{15,16}:

#	Label	Data Type	Period Type	Balance Type	Prefix	Standard label, Documentation, References, Concept name	Count						
1	Assets	Monetary	As Of (instant)	Debit	core	<p>Filer label: Assets</p> <p>Documentation: Assets are economic resources controlled by an entity.</p> <p>References:</p> <table border="1"> <thead> <tr> <th>Publisher</th> <th>Reference Name</th> <th>Reference Information</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation</td> </tr> </tbody> </table> <p>Name: core:Assets</p>	Publisher	Reference Name	Reference Information			URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation	1
Publisher	Reference Name	Reference Information											
		URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation											
2	Equity	Monetary	As Of (instant)	Credit	core	<p>Filer label: Equity</p> <p>Documentation: Equity is the ownership interest in the assets of an entity after deducting its liabilities.</p> <p>References:</p> <table border="1"> <thead> <tr> <th>Publisher</th> <th>Reference Name</th> <th>Reference Information</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation</td> </tr> </tbody> </table> <p>Name: core:Equity</p>	Publisher	Reference Name	Reference Information			URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation	1
Publisher	Reference Name	Reference Information											
		URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation											
3	Liabilities	Monetary	As Of (instant)	Credit	core	<p>Filer label: Liabilities</p> <p>Documentation: Liabilities are claims against assets by non-owners.</p> <p>References:</p> <table border="1"> <thead> <tr> <th>Publisher</th> <th>Reference Name</th> <th>Reference Information</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation</td> </tr> </tbody> </table> <p>Name: core:Liabilities</p>	Publisher	Reference Name	Reference Information			URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation	1
Publisher	Reference Name	Reference Information											
		URIDate: 2019-09-02 URI: https://en.wikipedia.org/wiki/Accounting_equation											

Three simple terms are defined: Assets, Liabilities, Equity.

¹⁵ Machine-readable terms, <http://xbrlsite.azurewebsites.net/2019/Core/core-ae/core.xsd>

¹⁶ Human-readable terms, <http://xbrlsite.azurewebsites.net/2019/Core/core-ae/evidence-package/contents/ReportElements-Concepts.html>

One functional term is defined: Balance Sheet:

#	Label	Prefix	Standard label, Documentation, References, Concept name	Count
1	Balance Sheet [Abstract]	core	<i>File label:</i> Balance Sheet [Abstract] <i>Documentation:</i> Elements of financial statements are the basic categories of items portrayed therein in order to meet the objective of financial statements. <i>References:</i> NONE <i>Name:</i> core:BalanceSheetAbstract	1

ASSOCIATIONS^{17,18}:

#	Label	Report Element Class	Period Type	Balance	Name
1	Core [Table]				(Implied)
2	Balance Sheet [Abstract]	[Abstract]			core:BalanceSheetAbstract
3	Assets	[Concept] Monetary	As Of	Debit	core:Assets
4	Liabilities	[Concept] Monetary	As Of	Credit	core:Liabilities
5	Equity	[Concept] Monetary	As Of	Credit	core:Equity

The association between the three terms and the balance sheet are provided.

ASSERTIONS^{19,20}:

#	Label	Result	Rule
1	Accounting Equation (Assets = Liabilities and Equity) (ASSERTION_CORE_Equality_AccountingEquation)	Pass	\$Assets = \$Liabilities + \$Equity

The mathematical relationship between the terms are established.

FACTS^{21,22, 23}:

#	Reporting Entity [Axis]	Period [Axis]	Concept	Fact Value	Unit	Rounding	Parentetical Explanations
1	GH259400TOMPUOLS65II (http://standards.iso.org/iso/17442)	2020-12-31	Assets	5000	USD	INF	
2	GH259400TOMPUOLS65II (http://standards.iso.org/iso/17442)	2020-12-31	Liabilities	1000	USD	INF	
3	GH259400TOMPUOLS65II (http://standards.iso.org/iso/17442)	2020-12-31	Equity	4000	USD	INF	

We can create a set of facts to exercise the logical system. Above assets of \$5,000, liabilities of \$1,000, and equity of \$4,000 were created.

¹⁷ Machine-readable associations, <http://xbrlsite.azurewebsites.net/2019/Core/core-ae/core-presentation.xml>

¹⁸ Human-readable associations, <http://xbrlsite.azurewebsites.net/2019/core/core-ae/evidence-package/contents/index.html#NetworkStructure-Core-Implied.html>

¹⁹ Machine-readable assertions, <http://xbrlsite.azurewebsites.net/2019/Core/core-ae/core-formula.xml>

²⁰ Human-readable assertions, <http://xbrlsite.azurewebsites.net/2019/Core/core-ae/evidence-package/contents/#BusinessRulesSummary.html>

²¹ Machine-readable facts, <http://xbrlsite.azurewebsites.net/2019/core/core-ae/instance.xml>

²² Human-readable facts, <http://xbrlsite.azurewebsites.net/2019/core/core-ae/evidence-package/contents/index.html#NetworkFacts-Core-Implied.html>

²³ Human-readable and machine-readable facts using Inline XBRL, <http://xbrlsite.azurewebsites.net/2019/core/core-ae/instance.html>

PRECISE AND COMPLETE:

The logical system can be called properly functioning because all of the statements are consistent with one another, there are no contradictions, it can be established that the logical system created precisely reflects the reality of the logical system and all the statements seem to be included within the logical system.

A software application can take all of the statements made within the machine-readable version of this logical system and perform work. Below you see a human-readable rendering of a balance sheet:

Balance Sheet [Abstract]	Period [Axis]
	2020-12-31
Balance Sheet [Abstract]	
Assets	5,000
Liabilities	1,000
Equity	4,000

MODEL:

Above we have shown one permissible interpretation or model of the logical theory. There is another permissible interpretation which exists but that we did not provide. That interpretation is that “Equity = Assets - Liabilities”. In this model the term “Equity” is often referred to as “Net Assets”.

Variability Caused by Alternative Intermediate Components

While financial reports must fit within the elements of a financial report defined by a financial reporting scheme; financial reports are not forms. Specific variability is anticipated and allowed by financial reporting schemes such as US GAAP, IFRS, IPSAS, GAS, FAS, etc.²⁴ By far, the most variability that exists within a set of financial statements exists on the income statement. SFAS 6 discusses the notion of intermediate components²⁵ of comprehensive income:

²⁴ Charles Hoffman, CPA, *Comparison of Elements of Financial Statements*, <http://xbrlsite.azurewebsites.net/2019/Core/ElementsOfFinancialStatements.pdf>

²⁵ FASB, SFAC 6, page 47, paragraph 77.

“Examples of intermediate components in business enterprises are gross margin, income from continuing operations before taxes, income from continuing operations, and operating income. Those intermediate components are, in effect, subtotals of comprehensive income and often of one another in the sense that they can be combined with each other or with the basic components to obtain other intermediate measures of comprehensive income.”

Variability can be caused by choosing to report different common subtotals, by choosing to report specific line items rather than others, by either providing or not providing a subtotal such as “Noncurrent assets”, etc. This variability is not random, arbitrary, or illogical; there are common patterns.

Here are common examples of variability using the accounting equation terms, association, and facts:

- **Add new term:** As mentioned, the term “Net Assets” is not provided in this model but is a permissible alternative to the term “Equity” for specific industries with which an economic entity is associated.
- **Add new association:** If a new term such as “Net Assets” is created, then a new association will always be necessary to relate the new term within some a structure to existing terms.
- **Change existing association:** If an existing association exists such as the association of “Equity” to “Balance Sheet”; it is possible to change that existing association and replace it with another permissible association.
- **Add new assertion:** If a new term is created it is likely that a new assertion will also tend to be necessary. For example, if the term “Net Assets” is created, the new assertion “Net Assets = Assets + Liabilities” would be created.
- **Change existing assertion:** Usually, existing assertions would not be changed but rather the existing assertion would not be used and would be replace by some other permissible assertion.
- **Add new structure:** An economic entity could decide to add an entirely new additional disclosure, creating a new structure that is used to articulate the associations between the terms that make up the new structure.
- **Change existing structure:** An economic entity could choose to modify the associations within an existing structure to provide a different but still permissible alternative disclosure.
- **Unreported fact:** An economic entity could choose to modify an existing structure to remove an association that provided an optional subtotal term and if so then the fact used to report that optional subtotal would not be provided within a report.

Note that “Change existing term” is not included in the list because it is never permissible to change the meaning of an existing term.

And so, the variability caused by these sorts of adjustments to some base model of a financial report must be managed and controlled in order be certain that the adjusted version of the model is still a permissible interpretation of a financial report and that the report is properly functioning.